Propagation of Electromagnetic Waves in a Space Charge Rotating in a Magnetic Field

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(Received August 20, 1941)

A theoretical discussion is presented of the transmission of electromagnetic waves of the symmetrical, transverse magnetic type by a space charge of uniform density, rotating under the influence of a uniform magnetic field. It is shown that the space charge presents to the wave an effective dielectric constant less than unity. By appropriate choices of magnetic field, the dielectric constant can be reduced to zero and can even become negative, so that the wave undergoes attenuation as it passes through the space charge. Experiments are described on the control of the propagation constants of 470-megacycle waves by variation of the dielectric constant of the rotating space charge medium. The results obtained are in agreement with the theoretical predictions.

INTRODUCTION

THIS paper presents the results of a theoretical and experimental study of the transmission characteristics of a uniform space charge rotating under the influence of a radial electric field and a uniform magnetic field. In a recent paper,[1] the authors presented a more general discussion of the high frequency behavior theoretically possible in such a space-charge medium. Normal modes of oscillation of such a space charge were deduced subject to various boundary conditions. The correspondence between the frequencies obtained with the observed frequencies of oscillation of the magnetron in the so-called "electronic mode" of oscillation, indicates that this discussion applies to this type of magnetron oscillation.

Also, close correspondence was found between the space-charge solutions and the solution of Maxwell's equations in free space.[2] The present study deals entirely with the propagation of waves of the transverse magnetic type having symmetry about the axis of rotation; these include the principal wave of the ordinary transmission line type and also the complementary or hyperfrequency E_0 type.

A simplified discussion of the theory is first given. It is shown that the d.c. condition of a space charge of uniform density and constant angular velocity in a uniform axial magnetic field and a radial electric field is consistent with the equations of motion and the field equations. Then, directly, from Maxwell's equations, it is seen that a symmetrical transverse magnetic wave of small amplitude will propagate in such a medium with the same distribution and velocity as in a free dielectric having a dielectric constant which is less than unity by a quantity proportional to the square of the magnetic field strength and inversely proportional to the square of the frequency. This effective dielectric constant may be reduced to zero or even become negative; for a magnetic field of 250 gauss, the dielectric constant passes through zero at 500 megacycles. As a medium for the propagation of waves, the variable dielectric permits control of phase velocity or propagation constant from near that of vacuum to imaginary values corresponding to exponential attenuations.

The experiments consisted in making the region of electronic dielectric a part of the dielectric of a coaxial transmission line. The inner conductor of the line served for part of its length as the cathode and the outer conductor as the anode. With loose coupling from an oscillator of fixed frequency to a section of tuned "electronic" transmission line, the shift in tuning point of a shorting disk on the line was observed for different values of the space charge parameters. The results obtained were in good agreement with the theoretical predictions.

[1] J. P. Blewett and S. Ramo, Phys. Rev. **57**, 635 (1940).
[2] Cf. J. R. Carson, S. P. Mead, and S. A. Schelkunoff, Bell Sys. Tech. J. **15**, 310 (1936).

The results suggest that a tube of the magnetron type might prove useful as a reactance tube in the ultra-high frequency range where distributed rather than lumped impedances are needed. The rotating space charge is of extremely high density, so as to give marked effects, with practically no current flow in the tube.

Theory

The solution of the d.c. field and force equations presented in the previous paper[1] was the same as that presented by A. W. Hull in 1924.[3] The charge density is given by

$$\rho_0 = eH_0^2/2\,mc^2, \tag{1}$$

where H_0 is the magnetic field strength, e and m are the charge and mass of the electron, c is the velocity of light. The angular velocity of the space charge is also constant and is given by

$$\Omega_0 = eH_0/2\,mc. \tag{2}$$

On this d.c. condition we superimpose a traveling wave in the axial or z direction, having electric field components in the z and r directions and a magnetic field component in the θ direction. This is the familiar transverse magnetic wave.

If the field components are

$$E_r e^{i(\omega t+\gamma z)}$$
$$E_z e^{i(\omega t+\gamma z)}$$
$$H_\theta e^{i(\omega t+\gamma z)},$$

the charge density is $\rho_0+\rho_1 e^{i(\omega t+\gamma z)}$ and the electron velocities are

$$v_r e^{i(\omega t+\gamma z)}$$
$$r\Omega_0 + v_\theta e^{i(\omega t+\gamma z)}$$
$$v_z e^{i(\omega t+\gamma z)}$$

in rational units; and if the a.c. components are small compared with the d.c. components, then from the force equations, the following relations exist between fields and velocities:

$$i\omega v_r - 2\Omega_0 v_\theta = \frac{e}{m}\left(E_r + \frac{H_0 v_\theta}{c}\right), \tag{3}$$

$$v_\theta = 0, \tag{4}$$

$$i\omega v_z = \frac{e}{m}E_z. \tag{5}$$

Substituting in (3) from (2).

$$i\omega v_r = \frac{e}{m}E_r, \tag{3'}$$

so that

$$\bar{v} = \frac{e}{im\omega}\bar{E}. \tag{6}$$

From the continuity condition

$$-\frac{\partial\rho}{\partial t} = \nabla\cdot(\rho\bar{v})$$

or

$$\rho_1 = (e\rho_0/m\omega^2)\nabla\cdot E. \tag{7}$$

We now substitute for $\bar{v}$ and ρ_1 in Maxwell's equations

$$\left.\begin{aligned} &\nabla\cdot\bar{H} = 0 \\ &\nabla\cdot\bar{E} = \rho \\ &\nabla\times\bar{H} = \frac{1}{c}\frac{\partial\bar{E}}{\partial t} + \frac{1}{c}(\rho\bar{v}) \\ &\nabla\times\bar{E} = -\frac{1}{c}\frac{\partial\bar{H}}{\partial t} \end{aligned}\right\} \tag{8}$$

to obtain

$$\left.\begin{aligned} &\nabla\cdot\bar{H} = 0 \\ &\nabla\cdot\left(1-\frac{e^2H_0^2}{2m^2\omega^2c^2}\right)\bar{E} = 0 \\ &\nabla\times\bar{H} = \frac{1}{c}\left(1-\frac{e^2H_0^2}{2m^2\omega^2c^2}\right)\frac{\partial E}{\partial t} \\ &\nabla\times E = -\frac{1}{c}\frac{\partial\bar{H}}{\partial t}. \end{aligned}\right\} \tag{9}$$

But these are the same as the equations for the propagation of waves through a dielectric medium of dielectric constant

$$\begin{aligned} \epsilon &= 1 - e^2H_0^2/2m^2\omega^2c^2 \\ &= 1 - 1.55\times10^{14}H_0^2/\omega^2 \\ &= 1 - (\lambda H/15{,}100)^2. \end{aligned}$$

[3] A. W. Hull, Phys. Rev. **23**, 112 (1924).

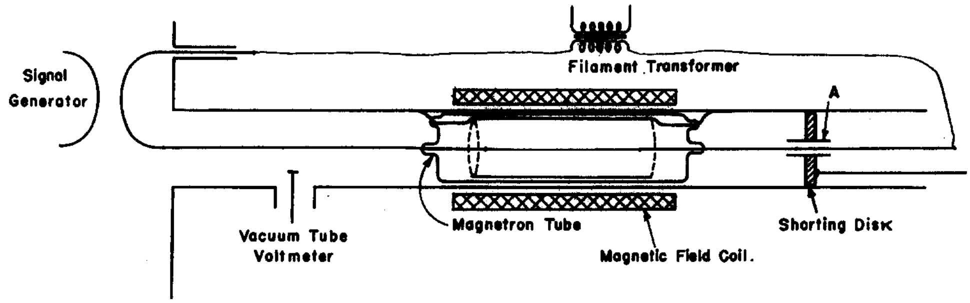

FIG. 1.

Thus it is evident that the rotating space charge will transmit transverse magnetic waves as though its dielectric constant were less than unity. This dielectric constant may be reduced to zero; for instance, at a frequency of 500 megacycles, the dielectric constant will disappear for a magnetic field of about 250 gauss. Higher fields will cause the dielectric constant to become negative. This reversal of sign of the dielectric constant will result in a change from real to imaginary values of the propagation constant γ. The product $i\gamma$ will become real and negative and the wave will attenuate as it travels through the space charge.

If a region of dielectric constant ϵ is inserted in a tuned transmission line and the line is again tuned to resonance, the new length will be $\sqrt{\epsilon}$ times the original length. The change in length will be proportional to $(1-\sqrt{\epsilon})$. This conclusion lends itself to experimental verification.

Experiments

The principle of operation is illustrated by Fig. 1, which represents an ordinary coaxial transmission line along part of whose length a rotating space charge is maintained. This is accomplished by making a tube of the magnetron type part of the line so that the cathode becomes a section of the inner conductor and the anode is sufficiently tightly coupled by capacity to the outer cylinder that it may be considered a part of the outer conductor of the line. A uniform magnetic field is maintained by a solenoid outside the line. The filament current of the magnetron is carried by the axial conductor, which is maintained at the required negative d.c. potential with respect to the outer conductor. The inner conductor is contained in a glass tube (not shown) for insulation, and tuning is accomplished by sliding a close fitting tube, A, over the glass tubing. This tube is attached to a shorting disk of conventional design, making the ensemble a tuned line. A vacuum tube voltmeter is located between the signal source and the magnetron tube in the region whose standing wave pattern should be unaffected by the operation of the tube and serves as a probe to indicate resonance of the line. For convenience in assembling, the outer conductor, in one case, was split up into semicylinders which were bolted together after the tube and inner conductor were in place. The outer conductor of another experimental line consisted of a set of telescoping tubes.

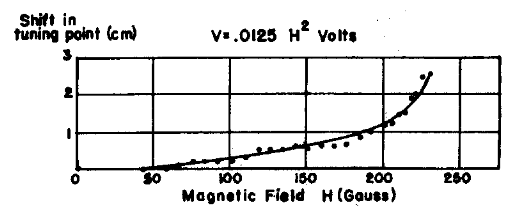

FIG. 2.

The tubes tested had anodes 6–10 cm long and 2–3 cm in diameter. The filaments were 10-mil tungsten wire. It was found that the spiral springs, which are usually included to maintain the tension of the filament, presented undesirable impedance effects, so the filaments were supported by molybdenum "zig-zag" springs of the type described by Tawney.[4]

The signal frequency was 470 megacycles. The line was sufficiently loosely coupled to the signal generator that variations in the properties of the line had no perceptible effect on signal frequency.

When the filament of the tube was heated, some decrease in maximum probe reading was

[4] G. L. Tawney, Rev. Sci. Inst. **10**, 152 (1939).

noted, presumably because of the increased resistance introduced in the inner conductor of the line.

The position of the shorting disk for resonance of the line was now measured as a function of magnetic field, anode voltage, and position of the tube with respect to the standing wave pattern of the line.

Figure 2 represents the shift in tuning point as a function of magnetic field. For this measurement, a constant ratio was maintained between the anode voltage and the square root of the

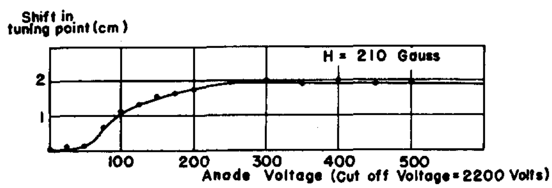

FIG. 3.

magnetic field so that the radius of the rotating space charge remained constant. The solid line is a plot of the theoretically expected function $L(1-(1-1.55\times10^{14}H^2/\omega^2)^{\frac{1}{2}})$ where L is the effective length of the space charge. Since end effects make it difficult to estimate this length, the ordinate of the curve was fitted at one point to the experimental relation. The maximum shift occurred at the field for which the theory predicts zero dielectric constant. No measurements could be made at higher fields because of the high attenuation of the wave as expected for negative dielectric constant. Due, presumably, to the non-uniformity of this structure, attenuation set in somewhat before the maximum shift in tuning point occurred.

Figure 3 represents the shift in tuning point with applied anode voltage. The magnetic field was held for these measurements at 225 gauss, approximately the field which gives zero dielectric constant. The voltage was increased from zero to the cut-off voltage of about 2200 volts, at which time the tube begins to pass appreciable current. As will be seen from Fig. 3, almost the whole shift takes place in the first 200 volts. This effect is to be expected, since even a very thin region of zero dielectric constant between the plates of a condenser will reduce its capacity to zero.

Figure 4 is a plot of shift in tuning point as the tube was moved with respect to the standing wave pattern of the transmission line. It is to be expected that the effect will be a maximum at a voltage loop, where the line is primarily a capacity; and a minimum at a voltage node, where the line is primarily inductive. This effect was observed as shown in Fig. 4. The asymmetry of the peak is again to be attributed to the combined effects of the asymmetry of construction and discontinuities introduced by the structure of the magnetron tube.

Application

The results presented above, besides supporting the theoretical predictions, suggest that a tube of the magnetron type may be very useful as a reactance tube at ultra-high frequencies. The use of an electron tube as a reactance is, of course, an established practice at low frequencies. In the ultra-high frequency range, however, conventional tubes possess undesirable transit time effects and interelectrode coupling. In addition, it is not practical to maintain in an electrostatic tube, a space charge sufficiently dense to modify, appreciably, the dielectric characteristics of the interelectrode space. The ro-

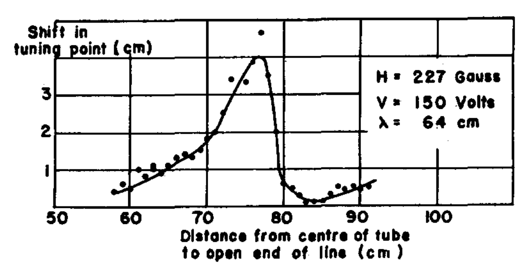

FIG. 4.

tating space charge does not appear to have these limitations. Construction of a practical reactance tube utilizing such a space charge will, of course, require further study to determine the structure which combines the high theoretical reactance with the lowest possible losses in the tube components.